DESIGNER'S GUIDE

op-amp feedback circuits
Part 2

Bode plots enhance feedback analysis of operational amplifiers

With a composite amplifier, a designer can selectively optimize the performance characteristics of both the individual amps and the overall circuit.

Bode plots graphically represent an op amp's gain, phase-margin, and noise characteristics. Part 1 of this series (Ref1) covered the feedback analysis of single-stage operational amplifiers; the conclusion examines the feedback behavior of composite op-amp circuits.

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Whereas the normal op-amp feedback loop involves only one amplifier, designers oftentimes need to extend the feedback loop to work with composite circuits that use two or more op amps for increased gain. By adhering to conventional feedback principles, you can implement phase compensation for the extended loop and rely on a Bode plot to provide a visual representation of the increased gain and the opportunity for extended bandwidth.

For instance, with two op amps in the same loop as in **Fig 1**, you can achieve increased gain without incurring any added offset and noise error. The input-error effects of the second amplifier are divided by the open-loop gain of the first amplifier. The net open-loop gain of this composite circuit becomes the product of the individual op-amp gains and greatly reduces the overall gain error and nonlinearity.

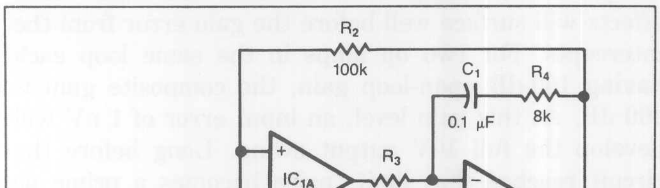

***Fig 1—To utilize the boosted gain of the composite amplifier,** traditional phase-compensation techniques tailor the gain-magnitude slope to obtain a stable region with a 1/β intercept.*

With a composite amplifier, a designer can selectively optimize the performance characteristics of both the individual op amps and the overall circuit.

In **Fig 1**, the two op amps are those of the dual OPA2111, which imposes only a modest cost increase over a single device. You could, of course, select individual op amps to provide specific performance characteristics. In the latter case, you might select the input amplifier for good dc and noise performance and the output amplifier for its load-driving and slewing performance. For example, the output amplifier could handle the load current and the resulting power dissipation, thus producing no thermal feedback to the input of the composite circuit. Moreover, it could also fulfill the high-slew-rate demands of the application. The input amplifier in this case would only swing through small signals.

Extending the dynamic range is feasible

An integrator and a common op-amp test loop can demonstrate the benefits of using a composite amplifier. By extending the composite open-loop gain to higher levels, you can expand the dynamic range for integrating analog functions. The low-frequency intercept moves back by a factor equal to the added amplifier gain. This change is so extreme that other error effects will surface well before the gain error from the intercepts. For two op amps in the same loop each having 100-dB open-loop gain, the composite gain is 200 dB. At this gain level, an input error of 1 nV will develop the full 10V output swing. Long before the circuit reaches that limit, noise becomes a prime ac constraint and typically restrains lower-level accuracy over a dynamic range of about 3,000,000:1. Nevertheless, this restraint is a 30:1 improvement over a single op amp and moves the focus of measurement accuracy to other factors.

In the op-amp test-loop application (**Ref 2**), the addition of the second amplifier removes signal swing from the output of the tested device. The extra gain transfers that voltage swing to an isolated output and removes any gain error from the signal detected at the tested amplifier's inputs. This gain-error removal permits the discernment of other input-error signals for the measurement of parameters such as power-supply and common-mode rejection. If these parameters were to approach the level of the amplifier's open-loop gain, gain-error signals at the input would cloud the effects of the measured parameter.

With a composite op-amp structure, you must include the roll-off characteristics of both amplifiers in your ac analysis and have some means of providing phase compensation for the loop. Two op amps in a common loop invite oscillation; the individual amplifier poles combine for a composite 2-pole roll-off. As shown in **Fig 1**, the logarithmic scale makes the initial composite-response curve the linear sum of the two individual responses. The upper, dashed response curve, which has a −40-dB/decade slope, shows this result.

Two methods are available for compensating the composite loop. One modifies the gain-magnitude response and the other alters the $1/\beta$ curve. The more usual of the two approaches is to reduce the slope of the gain-magnitude curve in the vicinity of the intercept—as **Fig 1** does. After forcing the compensated response to roll off earlier, the gain-magnitude curve returns with a more gentle slope to the boundary of the uncompensated response. This action serves the general-purpose requirements of voltage-gain applications and produces a stable range that you can place almost anywhere in the total composite-gain range.

Fig 1 achieves this compensation by creating a modified integrator response for IC_{1B}. Because this integrator is an inverting circuit, the inputs of IC_{1A} are reversed to retain only one phase inversion in the loop. Capacitor C_1 blocks the local dc feedback, and the overall gain is still the product of the two open-loop gains. The integrator response that R_3 and C_1 established for IC_{1B} rolls off this composite gain. Next, the first open-loop pole of IC_{1A} returns the compensated response slope to −40 dB/decade. At a higher frequency, a response zero provides the region of reduced slope thanks to the inclusion of R_4. Above the break frequency of R_4 and C_1, R_4 transforms the response of IC_{1B} from an integrator to an inverting amplifier with a gain of $-R_4/R_3$.

Where this gain is unity, the compensated response drops to and follows the open-loop response of IC_{1A} as shown. For gain levels other than unity, you have different options, which you can explore by using other response plots and defining the particular stable conditions you have in mind. Having control of this gain becomes particularly useful as the $1/\beta$ intercept approaches the uncompensated unity-gain crossover point. In this region, the second poles of the two op amps increase the phase shift. In such cases, you have to make the magnitude of the internal R_4/R_3 gain less than unity to force the compensated response to cross over earlier. Generally, when you have two op amps of the same type, making $R_4 = R_3/3$ will yield a unity-gain stable composite amplifier.

The net phase correction that you can achieve with this technique depends on the frequency-response range for which you maintain the −20-dB/decade slope. This span begins with the R_4C_1 break frequency and ends with the intercept of the composite open-loop response. After this intercept, the lack of open-loop gain returns the response to that of the uncompensated composite amplifier. To ensure a phase margin of 45° or more, you can use the guidance that the Bode phase approximation provides; the plot shows that this reduced slope region must last for three decades of frequency and must intercept the $1/\beta$ curve after running for at least a decade.

Extending the bandwidth may be desirable

Although most engineers are familiar with this type

higher gains, you can greatly extend the bandwidth and reduce the settling time by 40:1 by using a different phase-compensation technique. The general-purpose

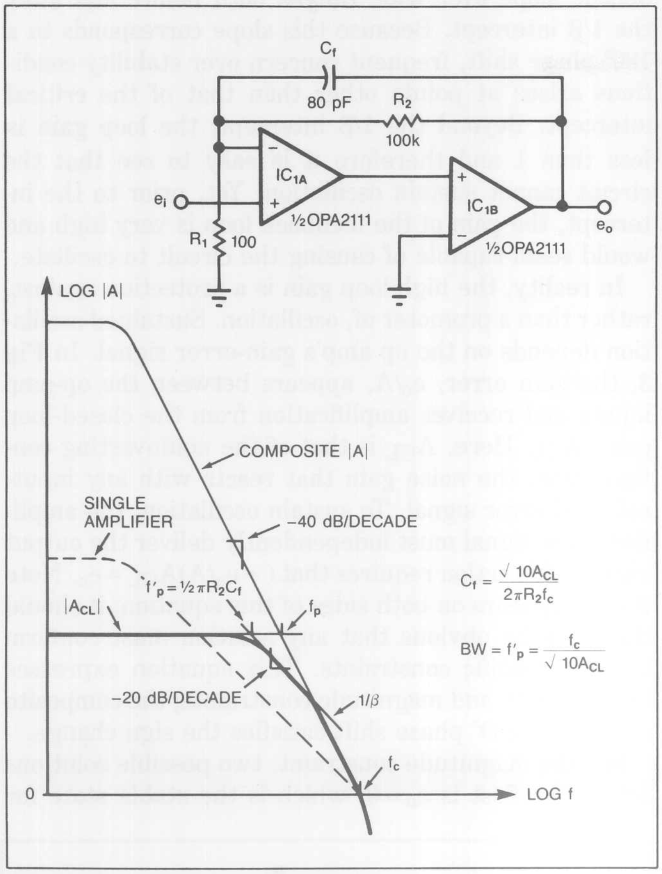

***Fig 2—For greater bandwidth in high-gain circuits,** you can provide phase compensation for the 1/β response to retain a smooth open-loop response for fast settling.*

$R_4 = R_3$ case of **Fig 1** sets a constant closed-loop gain-bandwidth product. Looking at the curves, you can see that the closed-loop bandwidth is the same as that for IC_{1A} itself when $BW = f_p = f_c/A_{CL}$. Even so, the large separation between the compensated and uncompensated responses shows a significant sacrifice in bandwidth—expressly for the accommodation of phase compensation. Uncompensated, the gain-magnitude response has a gain-bandwidth product that increases with closed-loop gain and that provides a potential bandwidth of $f_p = f_c/\sqrt{A_{CL}}$. Comparing the last two expressions shows that the potential for bandwidth improvement equals $\sqrt{A_{CL}}$, which is significant at higher gains.

Another way is to compensate the 1/β curve

You can take advantage of quite a bit of this bandwidth-improvement opportunity by compensating the 1/β curve instead of the gain-magnitude response curve. By referring back to the rate-of-closure stability criteria discussed in Part 1 (**Ref 1**), you would see that both curves contribute to the rate-of-closure parameter even though the gain-magnitude curve is generally the focus of phase-compensation efforts. To satisfy the rate-of-closure criteria regardless of the slopes of the individual curves. So, instead of reducing the gain-magnitude slope, increase the 1/β slope (**Fig 2**). A simple capacitive bypass of feedback resistor R_2 accomplishes this slope increase for a final 20-dB/decade rate-of-closure. An integrator configuration, on the other hand, with its special characteristics, inherently produces the −20-dB/decade slope for 1/β and achieves optimum bandwidth and dynamic range.

From a phase-shift perspective, this alternative approach to compensation adds phase shift to 1/β instead of subtracting it from the gain-magnitude response. It pushes the net phase shift beyond 180° toward 270°, rather than pulling it back toward 90°. In either case, the resulting phase margin approaches 90°, but the **Fig 2** method is simpler and achieves greater bandwidth. The pole created with C_f sets the bandwidth instead of the 1/β intercept.

Two factors distinguish this feedback-factor compensation technique for higher gains. Greater bandwidth is open for reclaiming, and the associated 1/β curves are well above the unity-gain axis. From higher levels, the 1/β roll-off is developed well before its intercept with the gain-magnitude curve. Starting this roll-off a decade ahead of the final intercept produces a 45° phase

Two methods are available for compensating the composite feedback loop. One modifies the gain-magnitude response, the other alters the 1/β curve.

adjustment for a like amount of phase margin. The slopes of the two curves show that, in order to accomplish this phase adjustment, C_f must break with R_2 one-half decade below the initial intercept frequency, f_p. Then, the 2:1 difference in slopes will place the new intercept one-half decade above f_p for the required full decade of the 1/β roll-off.

Time to brush up on equations

Again, the design equations for the required value of C_f and the resulting bandwidth are obvious from the logarithmic nature of the frequency axis. Setting f_p' at one-half decade below f_p implies that

$$\text{Log } f_p' = (\text{Log } f_p + \text{Log } f_p/10)/2,$$

for which $f_p' = f_p/\sqrt{10}$. From before, you'll remember that $f_p = f_c/\sqrt{A_{CL}}$ describes the uncompensated curve's bandwidth. The compensated bandwidth is

$$BW = f_p' = f_c/\sqrt{(10A_{CL})}.$$

Here, f_c is the unity-gain crossover frequency of the composite gain-magnitude response. As becomes obvious when you examine this expression, the improved bandwidth falls short of the total potential by $\sqrt{10}$. However, it is better than the **Fig 1** result by $\sqrt{(A_{CL}/10)}$, or a factor of 10, for a gain of 1000. Setting

C_f to break with R_2 at f_p' defines the value of this capacitor as

$$C_f = \sqrt{(10A_{CL})}/2\pi R_2 f_c.$$

For the op amps of the dual OPA2111 shown, the gain-of-1000 bandwidth becomes 20 kHz as compared with the 2 kHz you'd realize if you used just one of the op amps.

Improving the settling time

Settling time also improves when you choose the composite amplifier's $1/\beta$ curve for phase compensation. The improvement is a result of both the increased bandwidth and the retained constant gain-magnitude slope. For a single amplifier of the OPA2111 type, for a gain of 1000, the settling time would be 700 μsec to 0.01%. Because the **Fig 2** amplifier has 10 times the bandwidth of a single amplifier, the settling time drops by the same factor to 70 μsec. This improvement would not be possible without the smooth and continuous slope of the compensated-amplifier response. A response having an intermediate pole and zero such as **Fig 1** does has low-frequency response terms that are slow to settle following a transient. Known as an integrating frequency doublet, this pole/zero combination is notorious for its poor settling time **(Ref 3)**. By providing phase compensation for the $1/\beta$ curve, you ensure that the smooth gain-magnitude curve is left undisturbed, therefore achieving the optimum settling time.

At lower gains, the benefit of the $1/\beta$ compensation technique diminishes as does its control of phase. Because lower gains have $1/\beta$ curves closer to the unity-gain axis, they have less room for $1/\beta$ roll-off. To produce an intercept with the gain-magnitude curve after a decade of $1/\beta$ roll-off requires a minimum closed-loop gain of 10. Op-amp phase shifts impose further limits by growing from 90° to 135° as they approach the unity-gain crossover frequency. In the practical case, this phase-compensation method needs gains of 30 or more for good stability.

This type of phase compensation does have an unusual aspect: Too great a compensating capacitance will have a surprising effect. Whereas increasing such capacitance normally yields more damping and a more stable response, making C_f too large will cause instability. As C_f increases, the resulting intercept moves toward f_c and encounters the added phase shift of the secondary-amplifier poles. Even greater values of C_f will drop the $1/\beta$ curve to its limit at the unity-gain axis. From there, it proceeds along the axis to the magnitude-curve intercept that guarantees oscillation. Only a range of compensation-capacitor values provides stability with this second approach; the $1/\beta$ curves display this range for sensitivity-analysis purposes. Because of the capacitor's window of stable values, a random selection of C_f followed by a stability test is likely to miss the bandwidth opportunity of this technique.

Phase shift and stability come next

Another concept fundamental to op-amp feedback in composite-amplifier circuits becomes apparent when you examine phase shift and stability. Composite amplifiers such as the one in **Fig 3** produce a −40-dB/decade slope over wide ranges both before and after the $1/\beta$ intercept. Because this slope corresponds to a 180° phase shift, frequent concern over stability conditions arises at points other than that of the critical intercept. Beyond the $1/\beta$ intercept, the loop gain is less than 1 and therefore it is easy to see that the circuit cannot sustain oscillation. Yet, prior to the intercept, the gain of the feedback loop is very high and would seem capable of causing the circuit to oscillate.

In reality, the high loop gain is a protection against, rather than a promoter of, oscillation. Sustained oscillation depends on the op amp's gain-error signal. In **Fig 3**, the gain error, e_o/A, appears between the op-amp inputs and receives amplification from the closed-loop gain, A_{CL}. Here, A_{CL} is that of the noninverting configuration, the noise gain that reacts with any input-referred error signal. To sustain oscillation, the amplified error signal must independently deliver the output signal. This action requires that $(-e_o/A)A_{CL} = e_o$. Note that e_o appears on both sides of this equation; it should therefore be obvious that any solution must conform to very specific constraints. This equation expresses both polarity and magnitude constraints; the composite amplifier's 180° phase shift satisfies the sign change.

For the magnitude constraint, two possible solutions exist. The first is $e_o = 0$, which is the stable state for

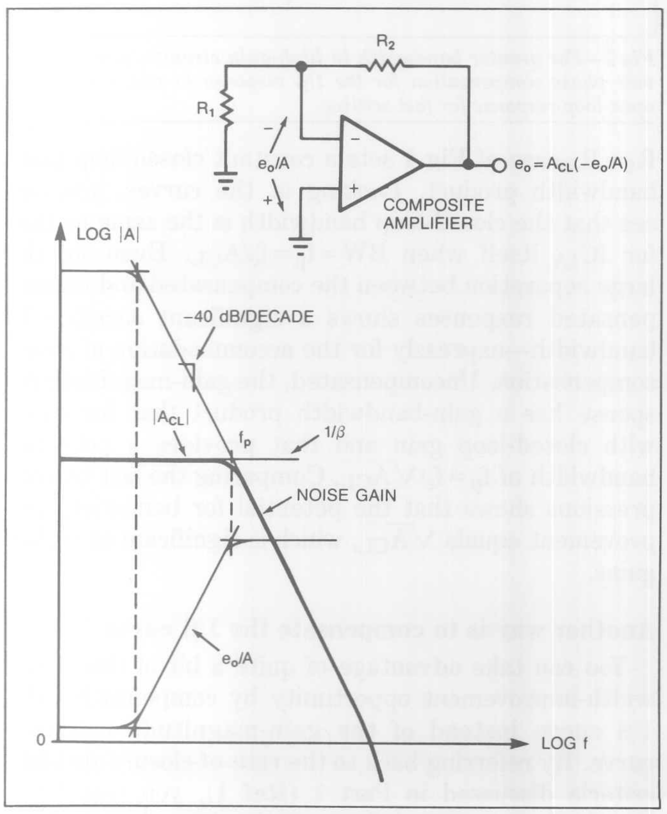

***Fig 3—A phase shift of 180° causes oscillation** only where the gain-error, e_o/A, is capable of independently supporting the output signal.*

the composite amplifier in the questioned region. There, the loop gain makes the signal e_o/A too small to independently support an output signal. In the plots of **Fig 3,** e_o/A starts at a very low level due to the high loop gain at low frequencies. As you move up in frequency, the gain-error signal rises while the amplifier-response slope signals its polarity inversion through the 180° phase shift. This inversion increases the output signal but cannot sustain it until the gain-error signal reaches a sufficient level. This critical level is a prerequisite for oscillation.

This level applies to the second solution for the magnitude constraint. At this level, A/A_{CL} has unity magnitude and maintains the balance for the previous feedback equation's magnitude requirement. Unity loop gain occurs at the $1/\beta$ intercept where the open-loop and noise-gain curves meet. Without phase-compensation intervention, this intercept satisfies both the phase and magnitude requirements for oscillation. Beyond this point, e_o and A fall off together, leaving the e_o/A signal constant and unable to support oscillation with the reduced gain. At the point where the magnitude of the gain error and the feedback phase shift must both reach specific levels to support oscillation, the intercept becomes critical. Before or after the intercept, the loop phase shift can be at any level and the gain-error magnitude will not be sufficient to cause instability.

Unfortunately, despite the composite amplifier's very specific requirements for oscillation, the greatly varied applications of op amps make this critical condition all too easy to encounter. To contend with this problem, you can rely on the $1/\beta$ curve to present a visual prediction of the problem and provide insight into a solution.

Variable feedback helps

Some applications demand that you include a second active element in the feedback loop to produce a varying feedback factor. In these applications, both the magnitude and the frequency characteristics of $1/\beta$ become variables. Fortunately, the gain- and feedback-response curves offer a means of quickly evaluating the range of conditions resulting from the changing feedback.

The most common way to provide magnitude variation in the feedback factor is to use a low-cost analog divider realization. Placing a multiplier in the feedback loop of an op amp **(Ref 4)** makes feedback a function of a second signal and therefore produces divider operation. With signal-dependent feedback, the bandwidth and stability conditions also become variables.

Various phase-compensation techniques are available to tailor the circuit's bandwidth and settling time.

Fig 4 shows the divider connection and demonstrates the effect of voltage-controlled feedback on $1/\beta$. The amplifier's feedback inverts the function of the multiplier by placing the feedback signal under the control of the e_2 signal. Then, the multiplier's transfer function of XY/10 delivers $e_o(e_2/10)$ to R_2. This action scales the feedback signal by comparing e_2 to a 10V reference level to obtain

$$\beta = (e_2/10)R_1/(R_1 + R_2).$$

With the feedback factor under control of this signal, the $1/\beta$ curve moves across the full range of the gain-magnitude response. As e_2 nears zero, the $1/\beta$ curve approaches infinity, leaving the op amp essentially in an open-loop configuration. At the other extreme, a full-scale 10V value for e_2 delivers a feedback signal to R_2 that equals e_o almost as if the multiplier were not present. Then, the net response is that of a simple

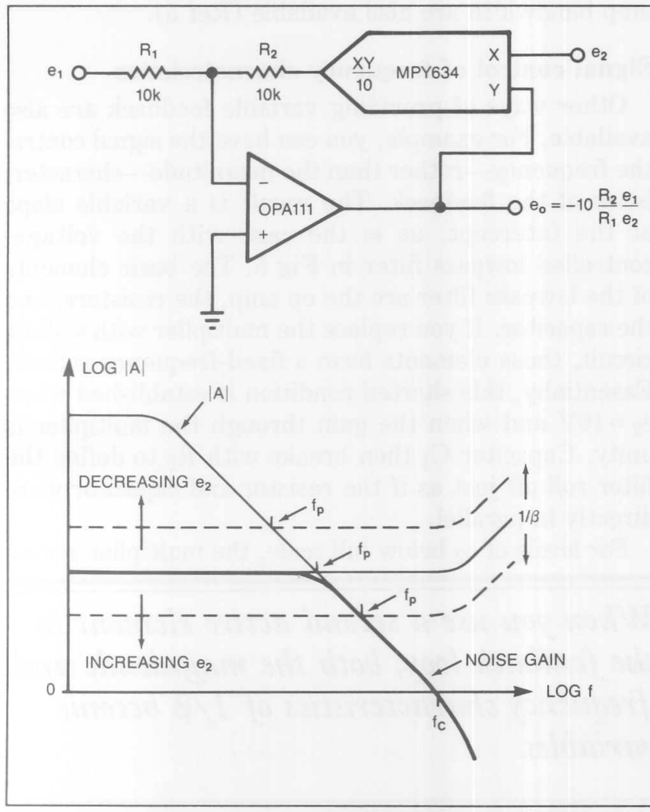

*Fig 4—**For this common analog divider,** a variable feedback magnitude allows a range of conditions that define bandwidth and stability.*

inverting amplifier with a feedback factor of $R_1/(R_1 + R_2)$ and an inverting gain of $-R_2/R_1$.

From one extreme to the other

Between the extremes, the variation of e_2 moves the $1/\beta$ curve from as low as the unity-gain axis to above the upper reaches of the amplifier's gain-magnitude curve. This variation moves the critical intercept and requires attention to the rate-of-closure over the entire span of the gain-magnitude response. If no significant multiplier phase shift exists, the feedback will always resemble that of an inverting amplifier for a zero $1/\beta$ slope, and you can ensure stability by just using a unity-gain-stable op amp. You can then read the range of bandwidth for the divider operation directly from the moving $1/\beta$ intercept. For a given

e_2 range, the intercept moves linearly with the signal, defining the corresponding bandwidth range.

The multiplier also introduces phase shift that alters the net phase shift around the feedback loop. Poles in the response of the multiplier circuit are zeros in the inverse $1/\beta$ function, causing the curve to rise at high frequencies. This rise moves toward the critical intercept when the multiplier control voltage, e_2, increases. This rise has an impact on the rate-of-closure, and the op amp must introduce a dominant pole to maintain stability. For the components shown, the OPA111 dominates the circuit roll-off because of its 2-MHz unity-gain crossover frequency. This frequency is well below the 10-MHz bandwidth of the MPY634 multiplier, placing the op amp in control. Other options that use a separate feedback path to restrict the op-amp bandwidth are also available (**Ref 5**).

Signal control of frequency characteristics

Other ways of providing variable feedback are also available. For example, you can have the signal control the frequency—rather than the magnitude—characteristics of the feedback. The result is a variable slope at the intercept, as is the case with the voltage-controlled lowpass filter in **Fig 5.** The basic elements of the lowpass filter are the op amp, the resistors, and the capacitor. If you replace the multiplier with a short circuit, these elements form a fixed-frequency roll-off. Essentially, this shorted condition is established when $e_2 = 10V$ and when the gain through the multiplier is unity. Capacitor C_1 then breaks with R_2 to define the filter roll-off just as if the resistor and capacitor were directly in parallel.

For levels of e_2 below full scale, the multiplier serves as a voltage-controlled attenuator to effectively alter the filter time constant. Attenuating the feedback voltage to R_2 lowers the signal current to the summing node, which has the same effect as increasing the resistor's value. Increased effective resistance corresponds to a decrease in the resistor's break frequency with C_1. This break defines the variable filter roll-off when

$$f_p = e_2/20\pi R_2 C_1.$$

When you use a second active element in the feedback loop, both the magnitude and frequency characteristics of $1/\beta$ become variables.

The maneuvering of the $1/\beta$ curve through this operation deserves closer inspection. The circuit exhibits a signal-dependent transition between the two different loops, which alternately controls the feedback. At low frequencies, C_1 is effectively an open circuit, and the controlling feedback path is through the op amp and the multiplier. This composite structure has resistive feedback that defines a signal gain of $-R_2/R_1$ and a noise gain of $(R_1+R_2)/R_1$. The latter relationship equals $1/\beta$ at low frequencies and the curve of interest

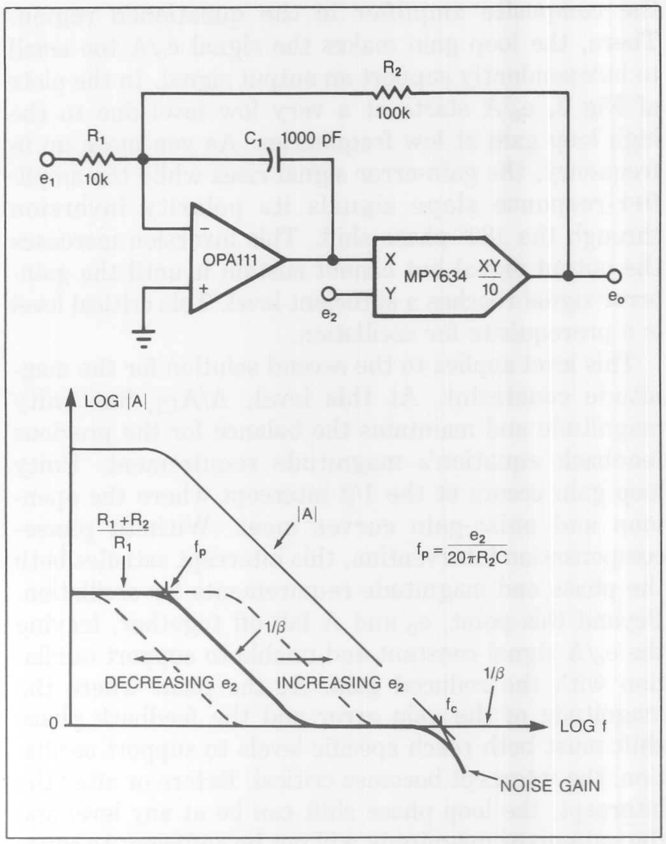

Fig 5—In this filter circuit, variations in the frequency characteristic of $1/\beta$ make possible a changing $1/\beta$ slope at the intercept.

starts at this level with a zero slope. At the high-frequency end, the composite structure is overridden when C_1 acts as a short circuit, which results in a unity feedback factor around the op amp. This short circuit absorbs all feedback current from R_2 without any corresponding change in the amplifier output voltage. The feedback loop of the composite structure is then disabled, switching feedback control to just the op amp. With C_1 then providing a unity feedback factor to the op amp, the $1/\beta$ curve follows the unity-gain axis at high frequencies.

Once the $1/\beta$ levels are fixed at the extremes, the multiplier determines the nature of the transition between the two. In the transition region, feedback currents from R_2 and C_1 compete for control of the summing node of the op-amp input. The contest for dominance is analogous to the frequency-dependent control of impedance with a parallel RC circuit. In both cases, the 3-dB point, where each element carries the same magnitude of current, defines the transition of control. The **Fig 5** filter achieves equal element currents when the impedance of C_1 and the effective impedance of R_2 are equal. This equality defines the voltage-controlled roll-off frequency of the filter as previously expressed. At this frequency, $1/\beta$ also rolls off and drops at -20 dB/decade to the high-frequency limit of the unity-gain axis.

What about controlling circuit stability?

The stability conditions of the **Fig 5** circuit depend on the particular feedback loop or the combination of

elements that are in control at the intercept point. For the lower-frequency filter cutoff frequencies illustrated, the op amp's bypass capacitor takes control before the intercept and defines the relevant feedback conditions. Because the $1/\beta$ curve follows the unity axis at the upper end, you can guarantee stability by ensuring that the op amp be unity-gain stable. For higher-frequency cutoff frequencies, the $1/\beta$ transition moves toward the gain-magnitude curve of the op amp. Circuit response cannot move beyond this limit, so the op-amp roll-off becomes the upper boundary of filter operation.

When the cutoff frequency approaches this boundary, the intercept rate-of-closure varies, prompting stability analysis. First, the zero of the $1/\beta$ curve approaches the intercept, where it increases the slope of the curve. Because this action reduces the rate-of-closure, stability is improved and a more detailed analysis is unnecessary. A continued increase in the cutoff frequency moves the $1/\beta$ curve further to the right where its pole interacts at the intercept. This break frequency returns the rate-of-closure to 20 dB/decade, thus retaining stability. Beyond this point, the intercept occurs at the flat lower end of the $1/\beta$ curve, and no further change in the rate-of-closure takes place.

Utilizing these various feedback conditions and a unity-gain-stable op amp, you can design a composite circuit that fulfills its primary stability requirement over the entire operating range. In addition, however, you may sometimes require a multiplier having a bandwidth much greater than that of the op amp, as the two previous examples demonstrate. Without a wide-bandwidth multiplier, $1/\beta$ would begin to rise near the higher-frequency intercepts and increase the rate-of-closure. The OPA111 avoids this complication when using the MPY634 multiplier by maintaining a dominant op-amp pole.

Other applications may involve feedback peaking and op amps that are not unity-gain stable—log amps and active filters, for example. For these and other variations requiring feedback analysis, the test remains the same. Look for the critical condition where the rate-of-closure is 40 dB/decade. Where conditions approach this level, conduct further analysis and compare phase-compensation alternatives for optimization.

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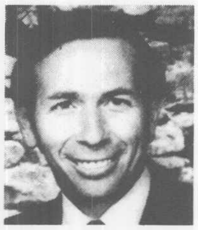
Author's biography



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